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Trend evaluation in records with long-term memory: Application to global warming

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[1] Previous statistical detection methods indicate that, on a global scale, the observed warming cannot be attributed solely to natural fluctuations. Here we estimate the probability $W(\Delta)$ that an observed trend Δ occurs naturally, and determine the anthropogenic part $A_Q(\Delta)$ of the temperature increase within a given confidence interval Q. To obtain these quantities, we do not use climate simulations, but assume as statistical null hypothesis that monthly temperature records are long-term correlated with a Hurst exponent $\alpha > 0.5$ (including also nonstationary records with α values above 1). We show that for confidence intervals with Q above 80% analytical expressions for $W(\Delta)$ and $A_Q(\Delta)$ can be derived, which request as input solely the Hurst exponent, as well as the temperature increase Δ obtained from the linear regression line and the standard deviation σ_t around it. We apply this approach to global and local temperature data and discuss the different results. **Citation:** Lennartz, S., and A. Bunde (2009), Trend evaluation in records with long-term memory: Application to global warming, *Geophys. Res. Lett.*, *36*, L16706, doi:10.1029/2009GL039516.

1. Introduction

[2] It is well accepted that the global mean surface air temperature has been rising in the 20th century, with a more pronounced increase in the last 50 years. The open question is how much of this increase can be attributed to natural fluctuations, and how much is of anthropogenic origin, caused, for example, by the increasing greenhouse gas (GHG) emission. This detection and attribution problem [*Bloomfield and Nychka*, 1992; *Hasselmann*, 1993; *Hegerl et al.*, 1996; *Zwiers*, 1999; *Barnett et al.*, 2005; *Rybski et al.*, 2006; *Zorita et al.*, 2008] plays an important role in the present climate debate. It has been recognized in the past decade that, due to processes occurring on the land surface, ocean, and cryosphere, the natural multidecadal temperature fluctuations are long-term correlated [*Koscielny-Bunde et al.*, 1998; *Pelletier and Turcotte*, 1999; *Weber and Talkner*, 2001; *Fraedrich and Blender*, 2003; *Monetti et al.*, 2003; *Rybski et al.*, 2008]. The expressions "long-term correlated", "long-term persistent" or "long-term memory", refer to time series, that are characterized by a Hurst exponent $\alpha > 1/2$. For continental land air temperature data α is around 0.65, while for sea-surface temperatures, α is in a broad region around 0.85 and also values above 1 cannot be excluded.

[3] In this letter we follow *Cohn and Lins* [2005], *Rybski et al.* [2006], *Zorita et al.* [2008], *Rybski and Bunde* [2009], and *Halley* [2009], and assume as null-hypothesis, that the natural temperature fluctuations are realizations of a long-term correlated process. As a consequence of the long-term memory pronounced persistent positive and negative deviations from the mean temperature occur, which leads to a clustering of extreme events [*Bunde et al.*, 2005]. On intermediate time scales, these deviations may look like trends [see, e.g., *Rybski and Bunde*, 2009], and this makes it difficult to distinguish between natural and anthropogenic temperature increases. Here we derive an analytical formula for the strength of a linear anthropogenic trend (within its 80% and 95% confidence levels) superimposed to the natural fluctuations. In addition, we show how significant changes in the anthropogenic trend can be detected, and apply our analysis to a large number of monthly local and global temperature records.

2. Data and Methods

[4] We have analyzed monthly land air, sea surface and combined temperatures of the northern and southern hemisphere, and the globe, provided by the Hadley Centre (http://www.cru.uea.ac.uk/cru/data/temperature/) [*Brohan et al.*, 2006]. In addition we have analyzed 30 local station monthly temperature records, kindly provided by the Potsdam Institute for Climate Impact Research (PIK). The Hadley-Centre CRU-data end in 2007, while the station records end around 1995 (see auxiliary material).[2] We focus on the last 50 and 100 years (if available). In each record we are interested in the Hurst exponent α and in the relative temperature increase during the considered period. To determine α, we use 2nd order detrended fluctuation analysis (DFA2 [see, e.g., *Kantelhardt et al.*, 2001]), where linear trends in the data are being removed systematically. To avoid seasonal effects, we have subtracted the seasonal mean value from the data and divided by the seasonal standard deviation. To determine the relative temperature increase in the considered time interval, we perform a linear regression analysis in the annual data, which yields the temperature change Δ and the standard deviation σ_t around the regression line. In contrast to the normal standard deviation σ around the mean value, σ_t is not effected by an external trend and thus represents the natural fluctuations. The dimensionless ratio Δ/σ_t is the relative temperature increase we are interested in. Figure 1 shows the results of this analysis for six representative climate records. The values of α, Δ and σ_t, estimated from Figure 1, are listed in the auxiliary material.

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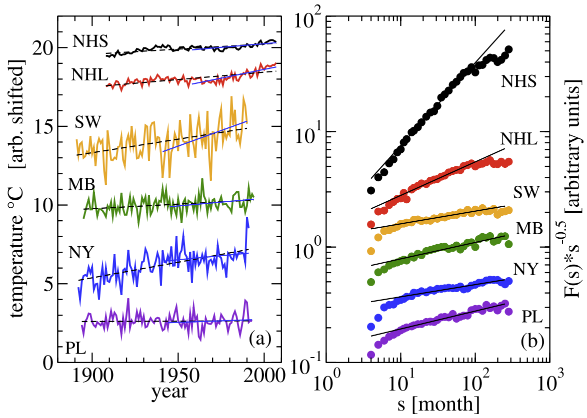

Figure 1. (a) From top to bottom: Annual temperatures of the northern hemisphere sea surface (NHS) (hadsst2nh), the northern hemisphere land (NHL) (crutem3nh), Swerdlowsk (SW), Melbourne (MB), New York (NY) and Plymouth (PL). The straight dashed lines are the linear regressions of the last 100 years and the straight full lines of the last 50 years. (b) DFA2 fluctuation function $F(s) \sim s^{\alpha}$ divided by $s^{0.5}$ for the same records (here monthly data). The curves have been shifted for clarity. The straight lines are the best fits between $s = 10$ and $s = 100$ months. The slopes α are (from top to bottom) 1.22, 0.79, 0.61, 0.64, 0.61 and 0.65.

[5] For estimating the probability that a certain relative temperature increase may occur due to natural fluctuations, we have studied long synthetic records with 90 α-values ranging from 0.41–1.30, generated by the standard Fourier-Filtering method. From these records we then extracted for each α-value 25.000 subsequences of length 600 and 1200, modeling the monthly temperature records of 50 and 100 years. In each subrecord we then determine Δ/σ_t and the local α-value by DFA2, which may differ from the global one.

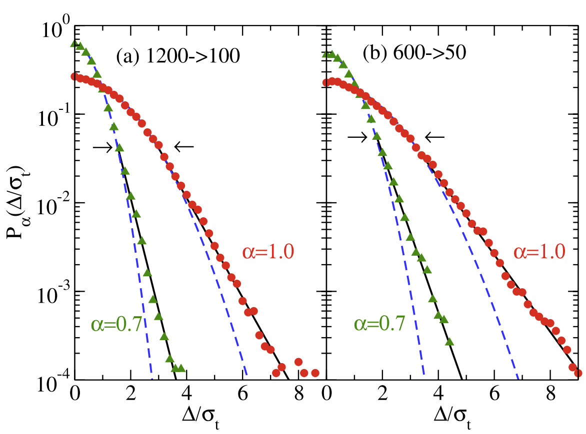

Figure 2. Probability density function $P_{\alpha}(\Delta/\sigma_t)$ of the relative temperature increase Δ/σ_t of synthetic long-term correlated records in the last (a) 100 and (b) 50 years. Before the trend estimation the monthly data have been averaged to annual data. The circles represent the probability density function for α = 1.0 and the triangles for α = 0.7. The dashed lines are Gaussians, which fit the data best for small arguments, while the straight lines are exponentials, which fit the data best for large arguments. The arrows indicate the crossover between Gaussian and exponential behavior.

3. Natural Occurrence Probability

[6] To obtain the probability density function (PDF) of Δ/σ_t, $P_{\alpha}(\Delta/\sigma_t)$, for fixed α-values in the relevant regime between 0.5 and 1.2, we have divided the local α-values into windows of size 0.02, and calculated in each window the normalized histogram of Δ/σ_t. Figure 2 shows the resulting $P_{\alpha}(\Delta/\sigma_t)$ in a semi-logarithmical plot for α = 0.7 ± 0.01 and α = 1.0 ± 0.01, both for 100 (Figure 2a) and 50 years (Figure 2b). Only positive values of Δ/σ_t are shown since P_{α} is symmetric in Δ/σ_t. Figure 2 shows that for small Δ/σ_t, the curves are Gaussian (dashed lines) with widths w_g, while for large Δ/σ_t, the curves follow a simple exponential (full lines). The crossover occurs roughly at $P_{\alpha} \simeq 0.05$. Figure 2 also shows that the PDFs depend sensitively on the size of the considered time window and on the Hurst exponent. From P_{α} we obtain, by direct summation, the probability $W_{\alpha}(\Delta/\sigma_t) = \int_{\Delta/\sigma_t}^{\infty} P_{\alpha}(x)dx$ that a relative trend greater than Δ/σ_t naturally occurs.

[7] Figure 3 shows that also W_{α} asymptotically decays exponentially (straight lines) for all α-values considered

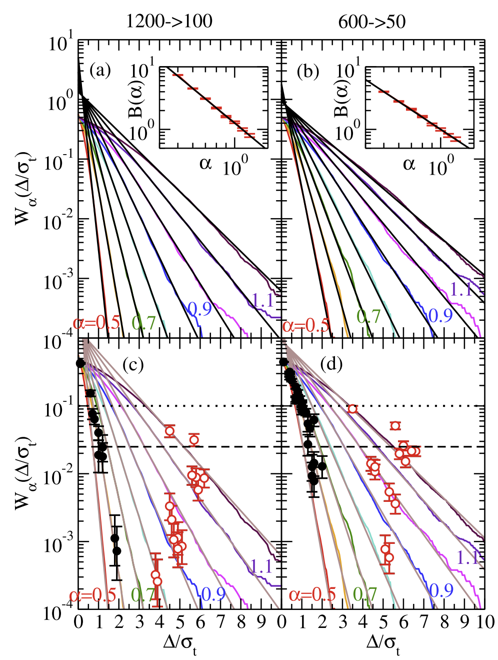

Figure 3. Cumulative probability $W_{\alpha}(\Delta/\sigma_t)$ (a and c) in 100 years, (b and d) in 50 years for Hurst exponents α between 0.5 and 1.2 (from left to right). The straight lines are exponentials with exponents $B(\alpha)$. The insets of Figures 3a and 3b show $B(\alpha)$ versus α, where the straight lines are the best fits to the data and error bars of ±0.05 were assumed. The full (open) symbols in Figures 3c and 3d represent the values we obtained for the local (global) records. We assumed that each α-value can be estimated up to an error of ±0.03. The dashed and the dotted lines represent the 95% and 80% confidence levels. Relative trends with probability below these values are considered as unnatural.

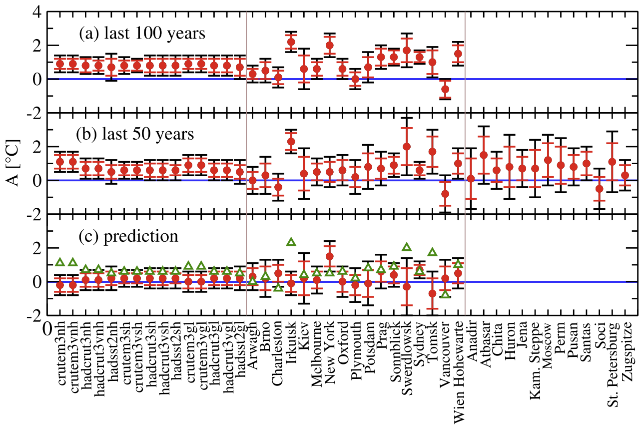

Figure 4. Measured temperature increase in the last (a) 100 and (b) 50 years of the records listed in the figure. The error bars refer to $A_{\min}(q)$ and $A_{\max}(q)$ from equations (4) and (5) for the 95% confidence interval (large error bars) and the 80% confidence interval (small error bars). (c) The circles represent the temperature increase in the first 50 years of the 100 year interval, which can be used for a prediction of the last 50 years. The error bars are roughly the same as in Figure 4b. The prediction can be compared with the actual temperature increase from Figure 4b, which are shown as triangles.

here. These straight lines meet at one point ($\Delta/\sigma_t \approx 0.2$), for both 50 and 100 years, suggesting

$$W_\alpha(x) = Ce^{-B(\alpha)(x-0.2)} \tag{1}$$

for $W_\alpha < 0.1$, with $B(\alpha) = D\alpha^{-\delta}$ (as shown in the inset of Figure 3). The constants are $C \simeq 0.8$, $D \cong 1.0$ and $\delta \cong 2.0$ for the 50 year period, and $C \simeq 1.15$, $D \cong 1.26$ and $\delta \cong 2.5$ for the 100 year period. According to equation (1), $W_\alpha(x)$ increases significantly with α. Thus, a given relative trend which is very unlikely for small α-values can become very likely for large α-values. The (less relevant) initial decay of W_α ($W_\alpha > 0.1$) can be described by the error function, see auxiliary material.

[8] Next we apply this formalism to the six records from Figure 1 for a 100 year time interval. For Plymouth (PL), New York (NY), Melbourne (MB), Swerdlowsk (SW), the northern hemisphere land (NHL) temperatures and the northern hemisphere sea (NHS) temperatures, we have $W_\alpha \simeq 0.43$, $<10^{-4}$, 0.025, 0.001, $3 \cdot 10^{-4}$, and 0.043, respectively. Accordingly, it is not likely that the relative 100 year trends of NY, SW and the NHL data are of natural origin. At the first glance, these results seem to be in line with the temperature increases obtained in Figure 1 which, for example, are larger in SW than in the NHS data. But the relative temperature increase in SW is smaller, suggesting that an anthropogenic trend is less pronounced. Since, however, the NHS data have a considerably higher Hurst exponent than the SW data, large relative trends are more likely to occur, and thus, even though Δ/σ_t is larger in the NHS data, the probability that it is of natural origin is enhanced.

[9] Except for NY, the greatest part of the temperature increase happened in the last 50 years, see Figure 1. One would thus expect that the anthropogenic signal is more significant in the last 50 years than in the last 100 years. But this is not the case. Except for PL, W_α increased considerably, for NY from below 10^{-4} to 0.13, for MB from 0.02 to 0.11, for SW from 0.001 to 0.013, for the NHL data from $3 \cdot 10^{-4}$ to $8 \cdot 10^{-4}$, and for the NHS data from 0.04 to 0.09. For PL, W_α decreased from 0.43 to 0.32, but in both cases the relative trend has a very high probability to be of natural origin. Accordingly, the modest temperature increase in the last 100 years is a stronger indicator for an anthropogenic trend than the stronger increase in the last 50 years. The results for all 45 records considered, listed in the auxiliary material and shown in Figure 3, confirm this finding.

4. Anthropogenic Trend

[10] In the following, we consider relative trends Δ/σ_t with occurrence probabilities in the $Q = (1 - 2q)$ confidence interval $q < W_\alpha < (1 - q)$ as natural. By definition, these relative trends are bounded by $\Delta/\sigma_t = -S_\alpha(q)$ and $\Delta/\sigma_t = S_\alpha(q)$ with $W_\alpha(S_\alpha(q)) = q$ and $W_\alpha(-S_\alpha(q)) = (1 - q)$. In most cases one considers $q = 0.025$, corresponding to a significance interval of 95%. In the following, we keep q (<0.1) and thus the size of the confidence interval general and obtain from equation (1),

$$S_\alpha(q) = 0.2 + \frac{1}{B(\alpha)} \ln(C/q). \tag{2}$$

If we add an external linear trend to the data that gives rise to a temperature increase of A°C in the considered time interval, both P_α and W_α are simply shifted by A/σ_t, and the significance interval is between $A/\sigma_t - S_\alpha(q)$ and $A/\sigma_t + S_\alpha(q)$. Accordingly, with probability $(1 - 2q)$ a measured value of Δ/σ_t is in the interval

$$\frac{A}{\sigma_t} - S_\alpha(q) \le \frac{\Delta}{\sigma_t} \le \frac{A}{\sigma_t} + S_\alpha(q). \tag{3}$$

These inequalities imply that with probability $(1 - 2q)$ the possible trend A is bounded by

$$A_{\min}(q) = \Delta - S_\alpha(q)\sigma_t \tag{4}$$

and

$$A_{\max}(q) = \Delta + S_\alpha(q)\sigma_t, \tag{5}$$

with $S_\alpha(q)$ given from equation (2). Equations (2), (4) and (5) solve the detection problem, since they allow to determine for any monthly data set of length 50 or 100 years an analytical estimation of the minimum and maximum anthropogenic trend, provided the temperature records can be considered as long-term correlated. If both $A_{\min}(q)$ and $A_{\max}(q)$ are above or below zero, this may be considered as a clear anthropogenic signal within a $(1 - 2q)$ confidence interval. Figures 4a and 4b show for all considered records $A_{\min}(q)$ and $A_{\max}(q)$ for $q = 0.025$ (large error bars) and 0.1 (small error bars) corresponding to a (conservative) 95% and a (less conservative) 80% significance interval, respectively. The results for the (very conservative) 99% confidence interval are discussed in the auxiliary material. Figure 4 confirms our finding in section 3, that the modest temperature increase in the last century is a stronger indicator for an anthropogenic trend than the considerably stronger increase in the last 50 years. Remarkable is the great uncertainty in the trend estimation which is (i) due to the long-term memory and (ii) due to large fluctuations σ_t around the regression line. These uncertainties are not an artifact of our approach, but a characteristic property of the complex climate system. For example, in the last 50 year interval, we see the most significant indication of an anthropogenic warming in Irkutsk, where $A_{\min}(0.025) \simeq 2$ and $A_{\max}(0.025) \simeq 3$. On the other hand, for St. Petersburg or Atbasar, where the upper bound $A_{\max}(0.025)$ is of the same order of magnitude as for Irkutsk, $A_{\min}(0.025)$ is negative, making an estimation of the present situation and of futural trends very difficult. The method also allows to test if there are significant changes in the trend between the first and the second half of the last century. To see this, we can predict the second 50 years on the basis of the first 50 years under the assumption that the trend is unchanged, and then check if the observed temperature increase is within the error bars of our prediction. Figure 4c shows the temperature increases Δ and their bounds $A_{\min}(q)$ and $A_{\max}(q)$ for the first 50 years of the 100 year records, which can be used as a prediction for the next 50 years. The actual temperature increase for each record is taken from Figure 4b and is indicated by triangles in Figure 4c. The figure shows that for the global records, 6 of 15 are inside the 80% interval, while for the local stations only 10 of 17 are within this interval. If the 95% confidence interval is considered, 11 of 15 global stations and 12 of 17 local stations are within this interval. For these records we cannot see a significant change in the trend.

5. Conclusion

[11] In this Letter, we used as null hypothesis that the natural temperature fluctuations are superimposed by an unknown linear anthropogenic trend, and can be modeled by long-term correlated data sets with a Hurst exponent $\alpha > 0.5$ (including also nonstationary records with α values above 1). This assumption is particularly appropriate for monthly data, where the short-term persistence, present in daily data, has been averaged out. We derived analytical expressions for the minimum and maximum trend in 50 and 100 years, $A_{\min}(q)$ and $A_{\max}(q)$, within a predefined arbitrary confidence interval $(1 - 2q)$. Solely three parameters, that are easy to extract from the data, are needed as input: (i) the Hurst exponent α obtained by standard DFA2, (ii) the temperature increase obtained by a linear regression analysis, and (iii) the standard deviation around the regression line. We applied this methodology to 15 global and 30 local stations. We found that in general, the trends in the global data are more significant than in the local data, an observation that has already been made by *Zorita et al.* [2008] when studying the probability that at least 13 of the warmest years in the past 127 years occurred in the last 17 years. Our analysis yields, in addition, the surprising result that the comparatively strong temperature increase in the last 50 years is a weaker indicator for an anthropogenic trend than the lower annual increase in the last 100 years. We have also used our approach to test for significant changes in the trend within the considered confidence interval. Our result yields only weak support for the thesis that the trend in the last 50 years changed its character compared with the first 50 years, since only 1/3 of the records show a remarkable change. Finally, our treatment cannot distinguish, by definition, between urban and global warming. Accordingly, the temperature increase that has to be attributed to global warming may be smaller in many of the local stations considered.

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